Based on K. H. Rosen: Discrete Mathematics and its Applications.

Lecture 22: Introduction to Probability. Section 7.1

1 Introduction to Discrete Probability

An **experiment** is a procedure that yields one of a given set of possible outcomes. The **sample space** of the experiment is the set of possible outcomes. An **event** is a subset of the sample space. Laplace's definition of the probability of an event with finitely many possible outcomes will now be stated.

Definition 1. If S is a finite nonempty sample space of **equally likely outcomes**, and E is an event, that is, a subset of E, then the probability of E is

$$\mathrm{P}(E) = \frac{|E|}{|S|}.$$

Remark 2. As a consequence of our definition we have, for any event E,

$$0 \leq \mathrm{P}(E) \leq 1.$$

An event E with probability $P(E) = 0$ is called **impossible**. An event E with probability $P(E) = 1$ is called **certain**.

Theorem 3. *(Probability of the complement) Let E be an event in a sample space S. The probability of the event $\bar{E} = S \setminus E$, the complementary event of E, is given by*

$$P(\bar{E}) = 1 - P(E).$$

Theorem 4. *(Probability of the union) Let E_1 and E_2 be events in the sample space S. Then*

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$

Remark 5. If the events E_1 and E_2 satisfy $P(E_1 \cap E_2) = 0$, the property above becomes

$$P(E_1 \cup E_2) = P(E_1) + P(E_2).$$

Events that satisfy $P(E_1 \cap E_2) = 0$, that is, events E_1, E_2 that **cannot occur simultaneously**, are called **mutually exclusive**.